INTERNET APPENDIX

Contents

A Proof of Lemma 1

Expected consumption is given by

$$(1-P)\, C_{bh} + pC_{bl} = M_1 + \theta^{SB} d^{SB} + \left(1 - p\left(1 - \bar{V}\right)\right) d^{TB} R^{TB}$$

when households do not withdraw their deposits early from traditional banks, and

$$C_b^w = M_1 + \theta^{SB} d^{SB} + d^{TB}$$

when they do.[1] Therefore, it is optimal for households not to withdraw their deposits under the condition[2]

$$\begin{aligned} (1-p)\, C_{bh} + pC_{bl} &\geq C_b^w \\ \therefore V \geq \bar{V} &\equiv \frac{1}{p}\left(\frac{1}{R^{TB}} - (1-p)\right) \end{aligned}$$

B Limited Liability

Consider a bank for which the limited liability constraint does not bind such that it borrows at the risk-free rate R and chooses $\{I_1, I_2, M_1, M_2\}$ to maximize expected profits

$$\max_{I_1, I_2, M_1, M_2} \quad (1-q)\left[\sigma_h I_1 + M_1\right] + q\left(\left[(1-p)\, \sigma_h + p\sigma_l\right] I_2 + M_2\right) - D$$

subject to (16), (18). Since $P_2 \leq (1-p)\, \sigma_h + p\sigma_l$ given $\phi \leq 1$, assets are priced at or below their expected payoff after bad news. Therefore the bank weakly prefers investing in I_2 over M_2 such that we can set

$$\begin{aligned} I_2 &= I_1 + \frac{M_1}{P_2} \\ M_2 &= 0 \end{aligned}$$

[1] Since households are atomistic, they do not internalize that their decision to withdraw deposits reduces the liquidation value of traditional banks.

[2] Note that, as long as households are risk neutral, all variables related to shadow banks drop out from the inequality. Therefore, $\bar{V}$ does not depend on the early withdrawal of shadow bank deposits.

This reduces the maximization problem to

$$\max_{I_1, M_1} \; I_1 + \left(1 - q + \frac{q}{\phi}\right) M_1 - \left(P_1 I_1 + M_1\right)$$

and the first order conditions for (I_1, M_1) are respectively

$$I_1 : P_1 = \frac{1}{1+\mu} \qquad (\mathcal{A}.1)$$

$$M_1 : \phi = 1 \qquad (\mathcal{A}.2)$$

where ($\mathcal{A}$.2) indicates that the bank will increase its cash holdings until $\phi = 1$.

Using these first order conditions, we can attain the following expression for profits in the state with low asset payoffs

$$\begin{aligned} \Pi_{bl} &= \sigma_l I_2 + M_2 - D \\ &= \left(\sigma_l - \frac{1}{1+\mu}\right) I_1 + \left(\frac{\sigma_l}{(1-p)\,\sigma_h + p\sigma_l} - 1\right) M_1 \end{aligned}$$

which indicates that the limited liability constraint binds in this state under the conditions

$$\sigma_l - \frac{1}{1+\mu} < 0$$

$$\frac{\sigma_l}{(1-p)\,\sigma_h + p\sigma_l} - 1 < 0$$

These conditions are respectively satisfied under (4) and $\sigma_l < \sigma_h$. Therefore, we prove by contradiction that limited liability binds in the state with low asset payoffs.

C Proof of Lemma 2

It follows from Appendix B that $V^{SB} < 1$. By combining (13) with (14), we can also show that $\bar{V} = 1$ such that failure to repay deposits fully leads to an early withdrawal.[3]

[3] Note that, although (13) and (14) are associated with traditional banks in Section 3.1.3, the conditions for a shadow bank to face an early withdrawal are identical under risk neutrality given expectations of no early withdrawal. In Appendix D, we also confirm that the optimal strategy followed by shadow banks given expectations of an early withdrawal leads to such a withdrawal.

D Proof of Lemma 3

The first order conditions to this problem indicate that shadow banks do not find it optimal to hold any cash $\left(M_1^{SB} = 0\right)$ when $R^{SB} > 1$.[4] Therefore, their liquidation value can be written as

$$\theta^{SB} = \frac{P_2}{P_1^{SB}} \tag{$\mathcal{A}$.3}$$

where P_1^{SB} is pinned down by the first order condition for the risky asset

$$P_1^{SB} = \frac{\sigma_h}{1+\mu}\frac{1}{R^{SB}} \tag{$\mathcal{A}$.4}$$

Combining ($\mathcal{A}$.4) and (15) yields

$$P_1^{SB} = (1-q)\,\frac{\sigma_h}{1+\mu} + qP_2 \tag{$\mathcal{A}$.5}$$

and by substituting this into ($\mathcal{A}$.3) we attain

$$\theta^{SB} = \frac{(1+\mu)\,P_2}{(1-q)\,\sigma_h + q\,(1+\mu)\,P_2} \tag{$\mathcal{A}$.6}$$

There will be a liquidity shortfall when $\theta^{SB} < 1$. Since θ^{SB} is increasing in $P_2 = \phi\,[(1-p)\,\sigma_h + p\sigma_l]$, setting $\phi = 1$ provides a sufficient condition for this. With some algebra, we can write this condition as

$$(1+\mu)\,[(1-p)\,\sigma_h + p\sigma_l] < \sigma_h$$

A further sufficient condition can be attained by setting the mark-up to its maximum value under $\omega = \underline{\omega}$. The condition then becomes $\sigma_h > 1$ which must be true.

To get an expression for interest rates, we combine ($\mathcal{A}$.6) with (15) such that

$$R^{SB} = \frac{1}{1 - q + q\frac{P_2}{\sigma_h}\,(1+\mu)}$$

[4] There is a no-short-sale constraint $(I_1, M_1) \geqq 0$ which is only binding for cash.

and $R^{SB} > 1$ follows from $\theta^{SB} < 1$. Substituting (16) and ($\mathcal{A}$.4) into (21) gives an expression for the expected payoff

$$E\left[\Pi^{SB}\right] = (1-q)\,\frac{\mu}{1+\mu}\sigma_h I_1^{SB}$$

where I_1^{SB} is attained by combining ($\mathcal{A}$.5) with the asset supply schedule (3) such that

$$I_1^{SB} = \frac{(A\alpha^{\alpha})^{\frac{1}{1-\alpha}}}{1+\omega}\left((1-q)\,\frac{\sigma_h}{1+\mu}+qP_2\right)^{\frac{\alpha}{1-\alpha}}$$

Observe that I_1^{SB}, P_1^{SB} and $E\left[\Pi^{SB}\right]$ are all positive related to P_2.

Finally, we confirm that the early withdrawal from shadow banks is optimal under this solution. It is optimal for households to withdraw their deposits early from shadow banks when

$$\left(1-p\left(1-V^{SB}\right)\right)R^{SB} < 1$$

where V^{SB} is defined according to (19). Using (15), ($\mathcal{A}$.3) and ($\mathcal{A}$.4), we can write

$$V^{SB} = (1+\mu)\,\frac{\sigma_l}{\sigma_h}$$

and the condition becomes

$$\frac{(1-p)\,\sigma_h + p\,(1+\mu)\,\sigma_l}{(1-q)\,\sigma_h + q\,(1+\mu)\,P_2} < 1$$

which is true under the restrictions $\phi > \underline{\phi}$, $\omega \geq \underline{\omega}$.

E Proof of Proposition 1

Combining (13) and (14) yields

$$R^{TB} = \bar{V} = 1$$

After substituting for $\left(R^{TB}, \bar{V}\right)$ and dropping the label 'TB' to simplify the exposition, the traditional bank's problem can be written in as

$$\Pi = (1-q)\left(\sigma_h I_1 + M_1\right) + q\,(1-p)\left(\sigma_h I_2 + M_2\right) - (1-qp)\,D \qquad (\mathcal{A}.7)$$

$$\text{s.t.}$$

$$P_1 I_1 + M_1 = D$$

$$P_2 I_2 + M_2 = P_2 I_1 + M_1 \qquad (\mathcal{A}.8)$$

$$\sigma_l I_2 + M_2 \geq D \qquad (\mathcal{A}.9)$$

$$(I_1, I_2, M_1, M_2) \geqq 0$$

where the last line represents no-short-sale constraints. There are three alternative cases depending on whether the no-withdrawal and no-short-sale constraint on I_2 bind. Below, we first describe the case in Proposition 1. We then detail the remaining two cases and prove that they may not be valid under the restrictions $\omega \geq \underline{\omega}$, $\phi > \underline{\phi}$.

E.1 Case 1

Proposition 1 describes the case where the no-withdrawal constraint ($\mathcal{A}.9$) and the no-short-sale constraint on I_2 bind. With $I_2 = 0$, the second period budget constraint ($\mathcal{A}.8$) and the no-withdrawal constraint can respectively be written as

$$\begin{aligned} M_2 &= P_2 I_1 + M_1 \\ M_2 &= P_1 I_1 + M_1 \end{aligned}$$

Therefore, the no-withdrawal constraint may only be satisfied with $I_1 > 0$ when

$$P_1 = P_2$$

which pins down P_1 and also corresponds to

$$I_1 = \frac{(A\alpha^{\alpha})^{\frac{1}{1-\alpha}}}{1+\omega} P_2^{\frac{\alpha}{1-\alpha}}$$

as per (3).[5]

Note also that the no-withdrawal constraint prevents the bank from converting M_1 to risky assets in the second period as long as $\phi > \underline{\phi}$. As such, the bank may not profit from holding cash in the first period and M_1 is indeterminate. Therefore, the expected payoff can be written as

$$\begin{aligned} E\left[\Pi^{TB}\right] &= (1-q)(\sigma_h - P_2) I_1 \\ &= (1-q)(\sigma_h - P_2) \frac{(A\alpha^{\alpha})^{\frac{1}{1-\alpha}}}{1+\omega} P_2^{\frac{\alpha}{1-\alpha}} \end{aligned}$$

and its derivative with respect to P_2 is

$$\frac{\partial E\left[\Pi^{TB}\right]}{\partial P_2} = \frac{(A\alpha^{\alpha})^{\frac{1}{1-\alpha}}}{1+\omega} \frac{1-q}{1-\alpha} \left(\frac{\alpha\sigma_h}{P_2} - 1\right) P_2^{\frac{\alpha}{1-\alpha}}$$

such that

$$\frac{\partial E\left[\Pi^{TB}\right]}{\partial P_2} < 0 \ \forall \ P_2 > \alpha\sigma_h$$

E.2 Case 2

In the first alternative case, the no-withdrawal constraint binds but the no-short-sale constraint is slack. By combining ($\mathcal{A}$.8) and ($\mathcal{A}$.9), we can write

$$\begin{aligned} I_2 &= \frac{P_2 - P_1}{P_2 - \sigma_l} I_1 \\ M_2 &= P_2 \left(\frac{P_1 - \sigma_l}{P_2 - \sigma_l}\right) I_1 + M_1 \end{aligned} \tag{$\mathcal{A}$.10}$$

where $P_2 > \sigma_l$ follows from $\phi > \underline{\phi}$ and M_1 is indeterminate as in Case 1. Substituting these into ($\mathcal{A}$.7) yields the following first order condition for I_1

$$P_1 = \frac{1}{1+\mu} \frac{(1-q)\sigma_h (P_2 - \sigma_l) + q(1-p)(\sigma_h - \sigma_l) P_2}{q(1-p)(\sigma_h - P_2) + (1-qp)(P_2 - \sigma_l)} \tag{$\mathcal{A}$.11}$$

[5] Any solution with $I_1 = 0$ is sub-optimal as (3) indicates that P_1 would approach zero.

and the expected payoff is

$$\Pi = \left[(1-q)\,\sigma_h + q\,(1-p)\,(\sigma_h - \sigma_l)\,\frac{P_2}{P_2 - \sigma_l} \right] \frac{\mu}{1+\mu} I_1 \tag{$\mathcal{A}$.12}$$

Finally, we derive a condition to eliminate this case by considering the no-short-sale constraint $I_2 \geq 0$. Since $P_2 > \sigma_l$, $I_1 > 0$, ($\mathcal{A}$.10) indicates that $I_2 \geq 0$ will bind when

$$P_2 < P_1$$

Using ($\mathcal{A}$.11), we can write this as

$$P_2 < \frac{1}{1+\mu} \frac{(1-q)\,\sigma_h\,(P_2 - \sigma_l) + q\,(1-p)\,(\sigma_h - \sigma_l)\,P_2}{(1-qp)\,(P_2 - \sigma_l) + q\,(1-p)\,(\sigma_h - P_2)} \tag{$\mathcal{A}$.13}$$

which implicitly establishes a boundary fire-sale discount $\hat{\phi}$ above which the no-short-sale constraint is slack.[6] Case 2 is eliminated for all $\phi \in [0,1]$ when $\hat{\phi} > 1$. The relevant condition can then be attained by combining ($\mathcal{A}$.13) with $\phi = 1$ such that

$$(1-p)\,\sigma_h + p\sigma_l < \frac{1}{1+\mu}$$
$$\therefore \omega > \underline{\omega}$$

which indicates that the restriction (4) eliminates Case 2.

Note also that even in the absence of the restriction (4), the equilibrium fire-sale never occurs under this case since the expected payoff ($\mathcal{A}$.12) is decreasing in ϕ. Without the restriction (4), the size of the shadow banking sector simply continues to expand until the equilibrium takes the form described in Appendix F with

$$\phi = \underline{\phi} \equiv \frac{(1-q)\,\frac{\sigma_h}{1+\mu} + q\sigma_l}{(1-p)\,\sigma_h + p\sigma_l}$$

[6] With $\sigma_l = 0$, we can get an explicit expression $\hat{\phi} = \frac{1}{1-q}\left(\frac{1}{1+\mu}\frac{1-qp}{1-p} - q\right)$.

E.3 Case 3

In the second alternative case, the no-withdrawal constraint is slack. Due to limited liability, banks strictly prefer to convert their cash to risky assets I_2 following bad news to profit from the decline in P_2. Therefore, we can write

$$\begin{aligned} I_2 &= I_1 + \frac{M_1}{P_2} \\ M_2 &= 0 \end{aligned}$$

and the first order conditions for (M_1, P_1) are respectively written as

$$\begin{aligned} P_1 &= \frac{\sigma_h}{1+\mu} \\ P_2 &< \sigma_h \end{aligned}$$

Since $P_2 < \sigma_h$ even without a fire-sale under bad news, banks optimally hold $M_1 \to \infty$. In other words, with the no-withdrawal constraint is slack, banks find it profitable to hold as much cash as possible in the first period and then convert all of it into risky assets after bad news. Since each unit of M_1 requires a unit of deposits, and risky assets contribute to low state revenues by $\sigma_l < 1$, it is impossible for the no-withdrawal constraint to remain slack under this investment strategy. Therefore, Case 3 is also eliminated.

F Solution under $\phi = \underline{\phi}$

Suppose $\phi < \underline{\phi}$ and hence $P_2 < \sigma_l$ such that traditional banks benefit from buying risky assets both in terms of profits and in terms of the no-withdrawal constraint. Therefore, they find it optimal to hold risky assets until the secondary market price returns to $P_2 = \sigma_l$ (i.e. $\phi = \underline{\phi}$). Let $\tilde{I}_2$ indicate the level of assets that achieve this, implicitly defined by the expression[7]

$$\frac{\sigma_l}{(1-p)\,\sigma_h + p\sigma_l} = f\left(\gamma I_1^{SB} + (1-\gamma)\left(I_1 - \tilde{I}_2\right)\right)$$

[7] We drop the label 'TB' to simplify the exposition

Once the secondary market price reaches $P_2 = \sigma_l$, the no-withdrawal constraint binds and traditional banks behave as described in 1. Therefore, I_1 is set according to $P_1 = P_2 = \sigma_l$ and traditional banks' holdings of safe assets in period 2 is given by

$$\begin{aligned} M_2 &= P_2 I_1 - P_2 \tilde{I}_2 \\ &= \sigma_l \left(I_1 - \tilde{I}_2 \right) \end{aligned}$$

where we have taken advantage of the indeterminacy of $M_1 \geq 0$ to set $M_1 = 0$. Finally, expected profits are given by

$$\begin{aligned} E\left[\Pi\right] &= \left(\sigma_h - \sigma_l\right) \left[\left(1-q\right) I_1 + q\left(1-p\right) \tilde{I}_2 \right] \\ &= \left(\sigma_h - \sigma_l\right) \left[\left(1-q\right) \frac{A^{\frac{1}{1-\alpha}} \left(\alpha \sigma_l\right)^{\frac{\alpha}{1-\alpha}}}{1+\omega} + q\left(1-p\right) \tilde{I}_2 \right] \end{aligned}$$

So far, we have assumed that traditional banks remain net-sellers with $\tilde{I}_2 \leq I_1$. When the excess supply of assets is particularly large, we may have $P_2 < \sigma_l$ even when traditional banks hold on to their risky assets such that $\tilde{I}_2 = I_1$. In this case, they will find it optimal to increase D and M_1 use this to purchase risky assets in period 2 until $P_2 = \sigma_l$. As before, the no-withdrawal constraint will bind at $P_2 = \sigma_l$ and the complete solution is

$$\begin{aligned} P_1 &= P_2 \\ I_2 &= I_1 + \frac{1}{\sigma_l} \tilde{M}_1 \\ M_2 &= 0 \\ D &= P_1 I_1 + \tilde{M}_1 = \sigma_l I_1 + \tilde{M}_1 \end{aligned}$$

where $\tilde{M}_1$ takes on the role of ensuring $P_2 = \sigma_l$ and is implicitly defined by

$$\frac{\sigma_l}{\left(1-p\right)\sigma_h + p\sigma_l} = f\left(\gamma I_1^{SB} - \left(1-\gamma\right) \frac{1}{\sigma_l} \tilde{M}_1 \right)$$

and the expected payoff is

$$E[\Pi] = (\sigma_h - \sigma_l)\left((1-qp)\frac{A^{\frac{1}{1-\alpha}}(\alpha\sigma_l)^{\frac{\alpha}{1-\alpha}}}{1+\omega} + q(1-p)\frac{1}{\sigma_l}\tilde{M}_1\right)$$
$$= (\sigma_h - 1)\left(\frac{1-qp}{qp}\frac{A^{\frac{1}{1-\alpha}}}{1+\omega}\left(\alpha\frac{1-(1-qp)\sigma_h}{qp}\right)^{\frac{\alpha}{1-\alpha}} + \frac{q(1-p)}{1-(1-qp)\sigma_h}\tilde{M}_1\right)$$

There are two notable implications. First, the secondary market price cannot go below σ_l. Second, a rise in the shadow banking sector size γ first leads to a rise in $\tilde{I}_2$, and then $\tilde{M}_1$. The above solution shows that $\tilde{M}_1$ and $E[\Pi]$ rise in this case while everything else stays constant. As we move to a limiting case with only shadow banks, safe asset holdings and traditional bank profits both approach infinity

$$\lim_{\gamma\to 1} E\left[\Pi^{TB}\right] = \lim_{\gamma\to 1}\tilde{M}_1 = \infty$$

which guarantees an inferior equilibrium for a sufficiently high commitment cost $\tau > \bar{\tau}$.

G Proof of Proposition 2

For the purposes of the proof, it is convenient to introduce some additional notation. Let ϕ^* denote the equilibrium fire-sale discount and the functions $\left(\pi^{SB}(\phi), \pi^{TB}(\phi)\right)$ map from the fire-sale discount to expected payoffs from shadow and traditional banking such that

$$\pi^{SB}(\phi) = (1-q)\frac{\mu}{1+\mu}\sigma_h\frac{(A\alpha^\alpha)^{\frac{1}{1-\alpha}}}{1+\omega}\left((1-q)\frac{\sigma_h}{1+\mu} + q\phi\left[(1-p)\sigma_h + p\sigma_l\right]\right)^{\frac{\alpha}{1-\alpha}}$$
$$\pi^{TB}(\phi) = (1-q)\left(\sigma_h - \phi\left[(1-p)\sigma_h + p\sigma_l\right]\right)\frac{(A\alpha^\alpha)^{\frac{1}{1-\alpha}}}{1+\omega}\left(\phi\left[(1-p)\sigma_h + p\sigma_l\right]\right)^{\frac{\alpha}{1-\alpha}}$$

as per Lemma 3 and Proposition 1. There is a mixed equilibrium when the following sufficient conditions are satisfied

$$\begin{aligned} \pi^{SB}\left(\overline{\phi}\right) &> \pi^{TB}\left(\overline{\phi}\right)-\tau && (\mathcal{A}.14)\\ \pi^{SB}\left(\underline{\phi}\right) &< \pi^{TB}\left(\underline{\phi}\right)-\tau && (\mathcal{A}.15)\\ \frac{\partial\pi^{SB}\left(\phi\right)}{\partial\phi} &> \frac{\partial\pi^{TB}\left(\phi\right)}{\partial\phi}\ \forall\ \phi\in\left(\phi^{*},1\right) && (\mathcal{A}.16)\\ \frac{\partial\phi}{\partial\gamma} &< 0\ \forall\ \gamma\in[0,1] && (\mathcal{A}.17) \end{aligned}$$

where $\tau>0$. In the sections below, we show that these conditions will be satisfied within a range of commitment costs $\tau\in(\underline{\tau},\bar{\tau})$ and also show that this range is non-empty.

G.1 Proof for condition ($\mathcal{A}$.14)

The condition depends on the value taken by

$$\bar{\phi}\equiv\min\left[1,\frac{(1-q)\frac{\sigma_h}{1+\mu}+q\sigma_l}{(1-p)\,\sigma_h+p\sigma_l}\right]$$

When we have

$$\mu<\frac{(p-q)\,(\sigma_h-1)}{qp\,(1-q)\,\sigma_h-(p-q)\,(\sigma_h-1)} \qquad (\mathcal{A}.18)$$

such that $\bar{\phi}=1$, the relevant condition is

$$\pi^{SB}(1)>\pi^{TB}(1)-\tau$$

Using the definitions for $\left(\pi^{SB}(.)\,,\pi^{TB}(.)\right)$, we can write this condition as a minimum commitment cost

$$\begin{aligned} \tau\geq\underline{\tau}\equiv(1-q)\,\frac{(A\alpha^{\alpha})^{\frac{1}{1-\alpha}}}{1+\omega}\,(\sigma_h-1)\left(\frac{1}{q}\right)^{\frac{1}{1-\alpha}}(1-(1-q)\,\sigma_h)^{\frac{\alpha}{1-\alpha}}\\ -(1-q)\,\frac{(A\alpha^{\alpha})^{\frac{1}{1-\alpha}}}{1+\omega}\,\frac{\mu}{1+\mu}\sigma_h\left(1-\frac{\mu}{1+\mu}\,(1-q)\,\sigma_h\right)^{\frac{\alpha}{1-\alpha}}>0 \end{aligned}$$

As an aside, we also show that $\underline{\tau} > 0$ such that a positive commitment cost is necessary for a mixed equilibrium. To do this, note that the expected payoff under Case 1 and Case 2 of the traditional bank's problem in Appendix E are equivalent when $\phi = 1$, $\omega = \underline{\omega}$. For any $\omega > \underline{\omega}$, profits under Case 1 are higher. Therefore, we can set $\omega = \underline{\omega}$, $\alpha < 0.5$ to write a sufficient condition

$$(1-q)\,\sigma_h\left[(1-q)\,\sigma_h + q\right] < 1$$

Note that the RHS is increasing in σ_h. A further sufficient condition is then to set $\sigma_l = 0$ which maximizes σ_h. We can then see that the above condition is true for all $p < 1$. Therefore, we can show that $\tau > 0$ under the two conditions

$$\begin{aligned} \alpha &< \frac{1}{2} \\ \omega &\geq \underline{\omega} \end{aligned}$$

When ($\mathcal{A}$.18) is not satisfied such that $\bar{\phi} < 1$, the relevant condition for ($\mathcal{A}$.14) is

$$\pi^{SB}\left(\bar{\phi}\right) > \pi^{TB}\left(\bar{\phi}\right) - \tau$$

which leads to a higher minimum commitment cost

$$\begin{aligned} \underline{\tau} \;=\; & (1-q)\,\frac{(A\alpha^{\alpha})^{\frac{1}{1-\alpha}}}{1+\omega}\left(\sigma_h - \bar{\phi}\left[(1-p)\,\sigma_h + p\sigma_l\right]\right)\left(\bar{\phi}\left[(1-p)\,\sigma_h + p\sigma_l\right]\right)^{\frac{\alpha}{1-\alpha}} - \\ & (1-q)\,\frac{(A\alpha^{\alpha})^{\frac{1}{1-\alpha}}}{1+\omega}\,\frac{\mu}{1+\mu}\sigma_h\left((1-q)\,\frac{\sigma_h}{1+\mu} + q\bar{\phi}\left[(1-p)\,\sigma_h + p\sigma_l\right]\right)^{\frac{\alpha}{1-\alpha}} \end{aligned}$$

where the aside on $\tau > 0$ is still valid.

G.2 Proof for condition ($\mathcal{A}$.15)

It follows from Appendix F that ($\mathcal{A}$.15) will be satisfied when the lower bound restriction on ϕ is violated such that

$$\underline{\phi} > f\left(\frac{(A\alpha^{\alpha})^{\frac{1}{1-\alpha}}}{1+\omega}\left((1-q)\frac{\sigma_h}{1+\mu}+q\sigma_l\right)^{\frac{\alpha}{1-\alpha}}\right)$$

for any $\tau < \infty$. Therefore, we do not necessarily need an upper bound on the commitment cost for a mixed equilibrium. However, the mixed equilibrium has different properties in the region $\phi < \underline{\phi}$ (as described in Appendix F) and we impose an upper bound on the commitment cost to prevent this.

Let $\widetilde{\phi} \equiv \frac{\alpha\sigma_h}{(1-p)\sigma_h+p\sigma_l}$ denote the fire-sale discount that maximizes traditional bank profits. The upper bound depends on where $\widetilde{\phi}$ stands relative to $\underline{\phi}$. When the following condition is true

$$\sigma_h \geq \frac{1}{1-qp(1-\alpha)} \qquad (\mathcal{A}.19)$$

such that $\widetilde{\phi} > \underline{\phi}$, the upper bound for commitment costs $\bar{\tau}$ must satisfy

$$\pi^{SB}\left(\widetilde{\phi}\right) < \pi^{TB}\left(\widetilde{\phi}\right) - \tau$$

which yields the upper bound

$$\tau \leq \bar{\tau} = (1-q)\frac{(\sigma_h A\alpha^{\alpha})^{\frac{1}{1-\alpha}}}{1+\omega}\left[(1-\alpha)\,\alpha^{\frac{\alpha}{1-\alpha}} - \frac{\mu}{1+\mu}\left(\frac{1-q}{1+\mu}+q\alpha\right)^{\frac{\alpha}{1-\alpha}}\right]$$

When ($\mathcal{A}$.19) is not satisfied, the fire-sale discount hits $\phi = \underline{\phi}$ before traditional bank profits peak and the upper bound is given by

$$\tau \leq \bar{\tau} = (1-q)\frac{(A\alpha^{\alpha})^{\frac{1}{1-\alpha}}}{1+\omega}\left[(\sigma_h-\sigma_l)\,\sigma_l^{\frac{\alpha}{1-\alpha}} - \frac{\mu}{1+\mu}\sigma_h\left((1-q)\frac{\sigma_h}{1+\mu}+q\sigma_l\right)^{\frac{\alpha}{1-\alpha}}\right]$$

Note that, regardless of the value taken by $(\bar{\tau}, \underline{\tau})$, it follows from ($\mathcal{A}$.16) that $\bar{\tau} > \underline{\tau}$.

G.3 Proof for condition ($\mathcal{A}$.16)

This follows directly from Lemma 3, which shows that

$$\frac{\partial \pi^{SB}(\phi)}{\partial \phi} > 0 \;\forall\; \phi \in (0,1)$$

and Lemma 1 which shows that

$$\frac{\partial \pi^{TB}(\phi)}{\partial \phi} < 0 \;\forall\; \phi \in \left(\widetilde{\phi}, \bar{\phi}\right)$$

where $\phi^* > \widetilde{\phi}$ since the latter is the peak of traditional bank profits in the range above $\underline{\phi}$.

G.4 Proof for condition ($\mathcal{A}$.17)

Recall that the excess supply of assets is given by

$$\widetilde{I} = \gamma I_1^{SB} + (1-\gamma)\left(I_1^{TB} - I_2^{TB}\right) > 0$$

where $\left(I_1^{SB}, I_1^{TB}\right)$ depend on ϕ and $I_2^{TB} = 0$. Given that $f(.)$ is continuous and decreasing, to satisfy ($\mathcal{A}$.17) we require that

$$\frac{\partial \widetilde{I}}{\partial \gamma} = I_1^{SB} - I_1^{TB} > 0 \;\forall\; \gamma \in [0,1]$$

which is equivalent to

$$I_1^{SB} > I_1^{TB} \;\forall \phi \in (\underline{\phi}, \overline{\phi}]$$

At any given ϕ, we have $I_1^{SB} > I_1^{TB}$ when the following condition is satisfied

$$q\sigma_h > (1+\mu)\,\phi\left[1-(1-q)\,\sigma_h\right]$$

Since the RHS is increasing in μ and ϕ, a sufficient condition is to set $\phi = 1$, $\mu = \bar{\mu}$, which will be satisfied for $\sigma_h > 1$.

Since $\left(I_1^{SB}, I_1^{TB}\right)$ both decrease in ϕ at different rates, we also need to show that I_1^{SB} at $\underline{\phi}$ is lower than I_1^{TB} at $\bar{\phi}$. This will be true with $\bar{\phi} = 1$ when ($\mathcal{A}$.18) is satisfied.

Otherwise, $\bar{\phi}$ will need to satisfy

$$\bar{\phi} = \frac{(1-q)\frac{\sigma_h}{1+\mu} + q\sigma_l}{(1-p)\sigma_h + p\sigma_l}$$

which is precisely how we define the upper bound restriction on the fire-sale discount.

G.5 Proof for the non-emptiness of $(\underline{\tau}, \bar{\tau})$

Finally, we prove that $\bar{\tau} > \underline{\tau}$ such that there is a non-empty set of commitment costs that bring about a mixed equilibrium. Since there are two alternatives values for both $\bar{\tau}$ and $\underline{\tau}$, we consider each in turn. First suppose that ($\mathcal{A}$.18) is satisfied so that $\underline{\tau}$ is in line with $\phi = 1$. Then it follows from Proposition 1 that $\bar{\tau} > \underline{\tau}$ regardless of which value $\bar{\tau}$ takes. Second, suppose ($\mathcal{A}$.18) is not satisfied so that $\underline{\tau}$ is in line with $\bar{\phi} < 1$. When ($\mathcal{A}$.19) is also not satisfied such that traditional bank profits do not peak until $\underline{\phi}$, $\bar{\tau} > \underline{\tau}$ follows from $\bar{\phi} > \underline{\phi}$.

The only case where we need to impose an additional condition corresponds to ($\mathcal{A}$.19) being satisfied so that $\bar{\tau}$ is in line with the peak $\phi = \frac{\alpha\sigma_h}{(1-p)\sigma_h + p\sigma_l}$ while ($\mathcal{A}$.18) is not satisfied such that $\bar{\phi} < 1$. The condition for non-emptiness is then equivalent to

$$\widetilde{\phi} < \bar{\phi}$$
$$\therefore \frac{\alpha\sigma_h}{(1-p)\sigma_h + p\sigma_l} < \frac{(1-q)\frac{\sigma_h}{1+\mu} + q\sigma_l}{(1-p)\sigma_h + p\sigma_l}$$

A sufficient condition is

$$\alpha < 1 - p$$

which should be satisfied when $\alpha < 0.5$, $p \leq 0.5$.

H Proof of Proposition 3

From the solution to the representative household's problem, we know that

$$\begin{aligned}\bar{V} &= 1-\frac{q}{p}\frac{\xi\left(1-\theta^{TB}\right)}{1-q\left(1-\xi\left(1-\theta^{TB}\right)\right)} \\ R^{TB} &= 1+\frac{q}{1-q}\xi\left(1-\theta^{TB}\right)\end{aligned}$$

With $\xi = 0$, these expressions simplify to $\bar{V} = R^{TB} = 1$. Note also that $\bar{V}$ is decreasing in ξ and R^{TB} is increasing. Therefore a rise in ξ leads to

$$\bar{V} < 1 < R^{TB}$$

Next, we show that the fire-sale on safe assets is a crucial determinant of liquidity risk. Consider the liquidation value given by (35). There will be no liquidity shortfall such that $\theta^{TB} = 1$ under the condition

$$P_2(r)\, I_2^{TB}(r) + P_2(s)\, I_2^{TB}(s) \geq D^{TB}$$

First, consider the case without a fire-sale on safe assets such that $P_2(s) = 1$. The condition becomes

$$I_2^{TB}(r) + I_2^{TB}(s) \geq D^{TB}$$

and will be true under any investment strategy that satisfies the no-withdrawal constraint (37) as long as

$$P_2(r) \geq \sigma_l$$

To see that $P_2(r) \geq \sigma_l$ must be true, consider what would happen otherwise. Since traditional banks are protected by limited liability, they do not internalize the state with weak fundamentals. Therefore, given $P_2(s) = 1 \geq P_2(r)$, traditional banks always prefer to purchase risky assets which yield a higher return in the state where they remain solvent. Ordinarily, traditional banks' risky asset purchases are limited by the no-withdrawal constraint. However, with $P_2(r) < \sigma_l$ and $P_2(s) = 1$, the strategy of selling a safe asset and purchasing risky assets with the funds increases the recovery rate V. Consequently, traditional banks increase their purchases of risky assets until their price rises to $P_2(r) = \sigma_l$.

Second, consider the case with a fire-sale on safe assets such that $P_2(s) = \phi$. Since shadow banks are liquidated after bad news, and traditional banks re-allocate their portfolio from risky to safe assets, there is an excess supply of risky assets at all times such that $P_2(r) = \phi$. We can then write the condition for $\theta^{TB} = 1$ as

$$\phi\left(I_2^{TB}(r) + I_2^{TB}(s)\right) \geq D^{TB} \tag{$\mathcal{A}$.20}$$

Note that the no-withdrawal constraint is not tightened by a decline in ϕ when both safe and risky assets are in excess supply, since the terms of trade between the two assets do not change, while the value of liquid asset holdings increase. Therefore, for sufficiently low ϕ, ($\mathcal{A}$.20) fails and there is a liquidity shortfall $\theta^{TB} < 1$.

I Example fire-sale function

To attain a simple fire-sale function from the outsider investor's problem, we can simply parameterize the payoff function from the outside investment to

$$g\left(\widetilde{K}\right) = \kappa^{-1}\ln\left(\widetilde{K}\right)$$

where $\kappa > 0$. The fire-sale function then becomes

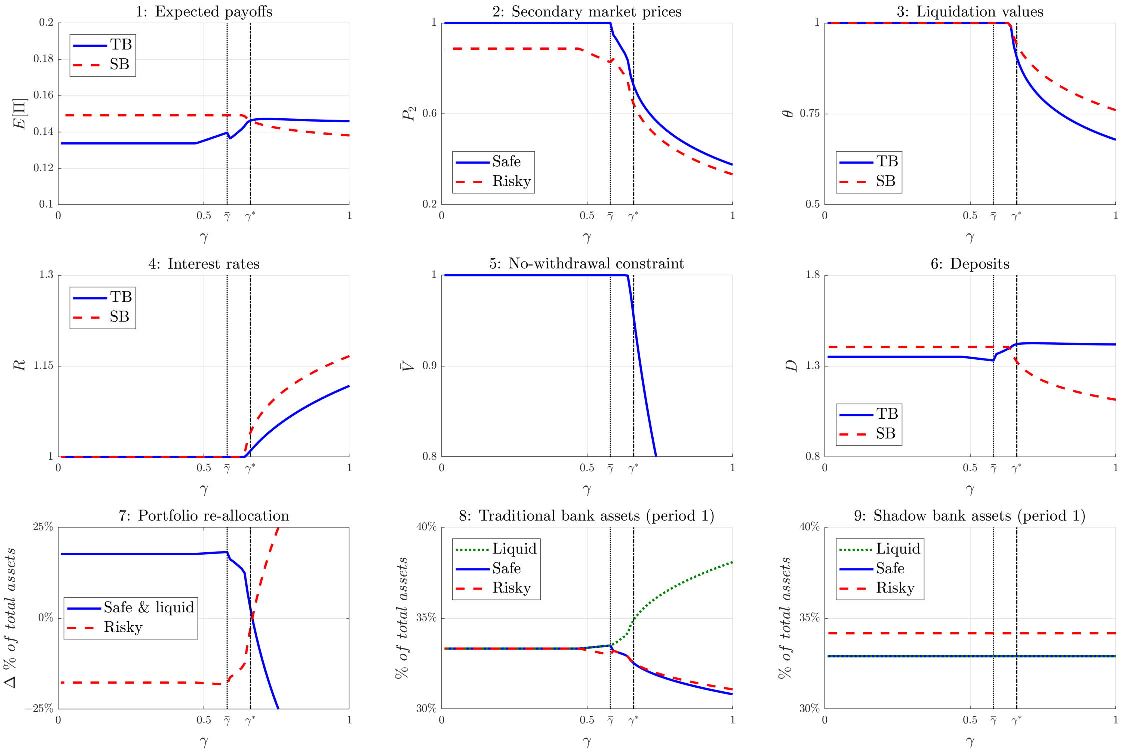
$$f\left(\widetilde{I}\right) = \frac{\kappa\widetilde{E}}{1 + \kappa\left[(1-p)\,\sigma_h + p\sigma_l\right]\widetilde{I}}$$

To satisfy the lower bound condition exactly, we need to set $\widetilde{E}$ at a level that yields $f\left(I_1^{SB}\right) = \underline{\phi}$ at $\gamma = 0$, which is

$$\widetilde{E} = \underline{\phi}\left[\frac{1}{\kappa} + (A\alpha^{\alpha})^{\frac{1}{1-\alpha}}\left((1-q)\frac{\sigma_h}{1+\mu} + \frac{1-(1-qp)\,\sigma_h}{p}\right)^{\frac{\alpha}{1-\alpha}}\right]$$

and the upper bound will approach but never exceed $\bar{\phi}$ as κ rises.

Figure 0.1: **Results under alternative bank-run specification**



Note: Expected payoffs are inclusive of the commitment cost τ. Total assets in period 1 and 2 are respectively defined as $\bar{I}_1 \equiv \sum_{i\in\{\lambda,s,r\}} I_1(i)$ and $\bar{I}_2 \equiv \sum_{i\in\{s,r\}} P_2(i)\, I_2(i)$. Panel 7 plots $\left(\bar{I}_2^{TB}\right)^{-1} P_2(s)\, I_2^{TB}(s) - \left(\bar{I}_1^{TB}\right)^{-1} \sum_{i\in\{\lambda,s\}} I_1^{TB}(i)$ for safe and liquid assets, and $\left(\bar{I}_2^{TB}\right)^{-1} P_2(r)\, I_2^{TB}(r) - \left(\bar{I}_1^{TB}\right)^{-1} I_1^{TB}(r)$ for risky assets. Panel 8 plots $I_1^{TB}(i)/\bar{I}_1$ respectively for $i = \{\lambda, s, r\}$ and Panel 9 does the same for shadow banks.

J Alternative specification for bank-runs

Following the global games solution of Goldstein and Pauzner (2005), we postulate that banks with a shortfall of liquidity are more vulnerable to bank-runs. Specifically, we depict the probability ξ that a bank faces a self-fulfilling run as a negative function $\zeta(.)$ of its liquidation value θ such that

$$\xi = \zeta(\theta),$$
$$\zeta'(.) \leq 0, \;\; \zeta(\theta) \in [0,1] \;\; \forall\, \theta$$

where $\zeta(1) = 0$ ensures that banks without a liquidity shortfall are not vulnerable to self-fulfilling runs.[8] We parameterize $\zeta(.)$ simply as

$$\zeta(\theta) = \max\left\{0, \min\left\{1, \widetilde{\xi}(1-\theta)\right\}\right\}$$

with $\widetilde{\xi} = 1.64$ calibrated in line with the calibration strategy described in Section 5.1. Figure 0.1 provides the numerical results under a set up and calibration that are otherwise identical to those presented in Section 5.2. In equilibrium, the two bank run specifications yield exactly the same set outcome. At above equilibrium sizes of shadow banking ($\gamma > \gamma^*$), the alternative bank run specification implies further increases in ξ in line with the decrease in liquidity. This leads to lower traditional bank profits and a sharper decline in minimum recovery rate $\bar{V}$ compared to the baseline case.

References

Goldstein, I. and Pauzner, A. (2005). Demand-deposit contracts and the probability of bank runs. *Journal of Finance*, 60(3):1293–1327.

[8]We also impose $\psi(\theta) \in [0,1]\ \forall\ \theta$ since ξ is a probability.